

Two dimensional QCD with matter in adjoint representation: What does it teach us?

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Abstract

We analyse the highly excited states in $QCD_2(N_c \rightarrow \infty)$ with adjoint matter by using such general methods as dispersion relations, duality and unitarity. We find the Hagedorn-like spectrum $\rho(m) \sim m^{-a} \exp(\beta_H m)$ where parameters β_H and a can be expressed in terms of asymptotics of the following matrix elements $f_{n_{\{k\}}} \sim \langle 0 | Tr(\bar{\Psi}\Psi)^k | n_k \rangle$. We argue that the asymptotical values $f_{n_{\{k\}}}$ do not depend on k (after appropriate normalization). Thus, we obtain $\beta_H = (2/\pi)\sqrt{\pi/g^2 N_c}$ and $a = -3/2$ in case of Majorana fermions in the adjoint representation. The Hagedorn temperature is the limiting temperature in this case.

We also argue that the chiral condensate $\langle 0 | Tr(\bar{\Psi}\Psi) | 0 \rangle$ is not zero in the model. Contrary to the 't Hooft model, this condensate does not break down any continuous symmetries and can not be considered as an order parameter. Thus, no Goldstone boson appears as a consequence of the condensation.

We also discuss a few apparently different but actually tightly related problems: master field, condensate, wee-partons and constituent quark model in the light cone framework.

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1. Introduction

Systems with a density of particle states increasing exponentially with energy

$$\rho(m) \sim m^{-a} \exp(\beta_H m) \quad (1)$$

have been of significant interest since the early days of the statistical bootstrap and dual models [1]. For such systems the canonical ensemble exists only for temperatures less than the Hagedorn temperature $T_H = \beta_H^{-1}$, because for $\beta < \beta_H$ the partition function $Z = \text{Tr} \exp(-\beta H)$ is divergent. One can think that at T_H one has a confinement - deconfinement transition in four-dimensional gauge theories with a confinement (hadron) phase at $T < T_H$ and a quark-gluon phase at $T > T_H$.

However one can ask whether the Hagedorn temperature is the limiting one or there is a phase transition in the system (1). As has been shown a long time ago by Frautschi and Carlitz [2], who considered both canonical and microcanonical ensembles for a gas of weakly interacting particles with a spectrum (1), the answer to this question depends on the numerical value of the parameter a in equation (1) - for $a > (D+1)/2$ one has a phase transition with canonical and microcanonical descriptions being not equivalent at high energies, while for $a \leq (D+1)/2$ one has T_H as a limiting temperature and canonical and microcanonical ensembles are equivalent. Here $D = d + 1$ is the dimension of space-time³.

To see what is the difference between a larger and smaller than $(D+1)/2$ let us consider the free energy of noninteracting particles with the mass spectrum (1). For bosons one gets

$$\begin{aligned} -\beta F_b = \ln Z_b &= -\frac{V}{(2\pi)^d} \int_{m_0}^{\infty} dm \rho(m) \int d^d p \ln \left[1 - \exp \left(-\beta \sqrt{p^2 + m^2} \right) \right] = \\ &= \frac{V}{(2\pi)^d} \int_{m_0}^{\infty} dm \rho(m) \sum_{n=1}^{\infty} \frac{1}{n} \int d^d p \exp \left(-n\beta \sqrt{p^2 + m^2} \right) \end{aligned} \quad (2)$$

and for fermions

$$\begin{aligned} -\beta F_f = \ln Z_f &= \frac{V}{(2\pi)^d} \int_{m_0}^{\infty} dm \rho(m) \int d^d p \ln \left[1 + \exp \left(-\beta \sqrt{p^2 + m^2} \right) \right] = \\ &= \frac{V}{(2\pi)^d} \int_{m_0}^{\infty} dm \rho(m) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \int d^d p \exp \left(-n\beta \sqrt{p^2 + m^2} \right) \end{aligned} \quad (3)$$

where m_0 is the infrared cut-off, which is usually of the same order of magnitude as β_H . Because we are looking at $\beta \sim \beta_H$ practically all particles can be

³Frautschi and Carlitz considered the case $D = 4$. The general case as well as $D = 10$ and/or $D = 26$ were considered in numerous papers in the mid-80's during the (super)string epoch. For review and references see, for example, [3]

considered as non-relativistic ones and we can rewrite $\exp(-n\beta\sqrt{p^2+m^2})$ as $\exp(-n\beta m - n\beta p^2/2m)(1 + O(n\beta p^4/m^3))$; it is easy to check that the neglected terms are of order $(n\beta m)^{-1} \ll 1$. Integrating over p we finally get

$$-\beta F_{b(f)} = \frac{V}{(2\pi)^{d/2}\beta^{d/2}} \int_{m_0}^{\infty} dm m^{d/2} \rho(m) \sum_{n=1}^{\infty} \frac{[1(-1)]^{n+1}}{n^{d/2+1}} \exp(-n\beta m) \quad (4)$$

Alternatively one can use the exact integral $\int d^d p \exp(-n\beta\sqrt{p^2+m^2}) \sim K_{D/2}(n\beta m)$ which leads to (4) for $m \gg \beta^{-1}$.

It is clear that in the vicinity of the Hagedorn temperature $\beta \rightarrow \beta_H$ one can neglect all terms in the sum with $n \geq 2$ (they will give singularities at $T = nT_H$) and there is no difference between bosons and fermions in the leading singular term with $n = 1$:

$$-\beta F = \frac{V}{(2\pi\beta)^{d/2}} \frac{1}{(\beta - \beta_H)^{d/2+1-a}} \Gamma\left(\frac{D+1}{2} - a, (\beta - \beta_H)m_0\right) \quad (5)$$

+less singular terms

where $\Gamma(x, y) = \int_y^{\infty} dt t^{x-1} e^{-t}$ is an incomplete Γ function.

Now one can see that for $(D+1)/2d/2 + 1 > a$ the free energy has a power singularity when $\beta \rightarrow \beta_H$, whereas in the case $a > (D+1)/2$ it is finite at $\beta = \beta_H$. For $a = (D+1)/2$ one has a logarithmic singularity $\beta F \sim \ln((\beta - \beta_H)m_0)$. Thus in the case $a < (D+1)/2$ T_H is the limiting temperature - no matter how much energy we put into system its temperature will be less than T_H , contrary to the case $a > (D+1)/2$ where the energy density is finite at $T = T_H$ and one has a phase transition point.

The only known examples of spectra where the parameters a are known analytically are the critical bosonic and fermionic strings at $D = 26$ and $D = 10$ respectively. It is known (see [3] for references) that in both cases one has $a_{closed} = D > (D+1)/2$ for closed and $a_{open} = (D-1)/2 < (D+1)/2$ for open strings, i.e. the Hagedorn temperature is a limiting temperature for open strings.

In this paper we shall consider the spectrum in a $1+1$ dimensional QCD gauge theory with Majorana fermions in the adjoint representation with the action

$$S_{adj} = \int d^2 x Tr \left[-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \gamma^\mu D_\mu \Psi + m\bar{\Psi} \Psi \right] \quad (6)$$

The light-cone quantization of this theory was considered in a large N_c limit in [4]. The spectrum consists of closed-string excitations. Contrary to the 't Hooft model [5] with fermions in the fundamental representation of $SU(N_c)$ describing the open-string excitations with only the meson Regge trajectory, in this theory there is an infinite number of closed-string Regge trajectories and the density of particle states is of type (1) [7], [6]. The numerical value of

$\beta_H \approx (0.7 - 0.75)\sqrt{\pi/(g^2 N_c)}$ was found in the large N_c limit in [6]. The same picture was obtained in [8] for a 1 + 1- dimensional QCD with adjoint scalar quarks. The numerical value of the inverse Hagedorn temperature in this case is $\beta_H \approx (0.65 - 0.7)\sqrt{\pi/(g^2 N_c)}$. Recent progress in studying the spectrum in the light cone gauge was reported in [9], [10]. However it was impossible to extract any information about the parameter a using numerical methods. It is the main goal of this paper to calculate both β_H and a analytically using something like quark-hadron duality in a two-dimensional world.

This paper is organized as follows. In Sect.2 we use very general methods (like dispersion relations and duality) in order to extract information on highly excited states in $QCD_2(N \rightarrow \infty)$ (exactly these states determine the Hagedorn temperature).

In Sect.3 we establish the connection between our results (based on very general ideas) and previous numerical calculations. We argue that both results agree with each other if the theory possesses the condensate $\langle 0|Tr(\bar{\Psi}\Psi)|0\rangle$. We argue that the nonzero magnitude for the condensate is related somehow to the master field at large N_c . However, an explicit realization of this connection is still lacking. We also discuss the relation between the condensate $\langle 0|Tr(\bar{\Psi}\Psi)|0\rangle$ and the phenomenon of mixing of components with different parton numbers. The corresponding analysis raises another (but related) problem about an infinite number of constituents near zero momentum (so called wee partons). We believe that all these problems are tightly connected to each other and $QCD_2(N \rightarrow \infty)^{adj}$ is the perfect toy model to elaborate these relations.

Sect.4 is our Conclusion and outlook.

2. The spectrum of $QCD_2(N_c \rightarrow \infty)$.

First of all, we would like to recall an example of two dimensional QCD coupled to fundamental matter [5], where the dispersion relations and duality, in turn, are very powerful tools. Many interesting results can be obtained by using these methods. The main idea is to relate the known spectrum of QCD_2 to the different vacuum characteristics. The same methods are very useful for the study of very general properties of the spectrum itself.

In particular, in the weak coupling regime, $N_c \rightarrow \infty$, $g^2 N_c \sim const.$, $g \ll m_q$ the chiral condensate $\langle \bar{\psi}\psi \rangle$ at $m_q \rightarrow 0$ has been calculated *exactly* [11]. The original calculation was based on dispersion relations and duality. The result was confirmed by numerical [12],[13] and independent analytical calculations[14]. Moreover, the method has been generalized for nonzero quark mass and the corresponding explicit formula for the chiral condensate $\langle \bar{q}q \rangle$ with arbitrary m_q has been obtained [15]. Let us note that there is no contradiction with the Coleman theorem [16] at this point, because the BKT (Berezinski-Kosterlitz-Thouless)[18] behavior takes place in the large N limit[11].

Furthemore, the so-called mixed vacuum condensates with arbitrary number of gluon insertions, $\langle 0|\bar{q}(g\epsilon_{\mu\nu}G_{\mu\nu})^nq|0\rangle \sim M_{eff}^{2n}\langle 0|\bar{q}q|0\rangle$, have also been calculated [19]. We interpret the factorization property for the mixed vacuum condensates as reminiscent of the master field at large N .

Besides that, some low-energy theorems can be obtained and from this we can see that there are no other states in addition to the ones found by 't Hooft. In other words, if we miss some states the dispersion and duality relations would indicate this.

Let us demonstrate how it works in the simplest case and consider the asymptotic limit $Q^2 = -q^2 \rightarrow \infty$ of the two-point correlation function [17], [11]:

$$i \int dx e^{iqx} \langle 0|T\{\bar{\psi}i\gamma_5\psi(x), \bar{\psi}i\gamma_5\psi(0)\}|0\rangle = \Pi(Q^2). \quad (7)$$

It is clear, that the large Q^2 behavior of $\Pi(Q^2)$ is governed by the free, massless theory, where

$$\Pi(Q^2 \rightarrow \infty) = -\frac{N_c}{2\pi} \ln Q^2. \quad (8)$$

At the same time the dispersion relations state that

$$\Pi(Q^2) = \frac{N_c}{\pi} \sum_{n=0,2,4,\dots} \frac{f_n^2}{Q^2 + m_n^2} \quad (9)$$

and the sum is over states with an even number n (n is the excitation number) because we are considering states with given parity. Here the matrix elements f_n are defined as follows

$$\langle 0|\bar{\psi}i\gamma_5\psi|n\rangle = \sqrt{\frac{N_c}{\pi}} f_n, \quad n = 0, 2, 4, \dots \quad (10)$$

Bearing in mind that for large n , $f_n^2 \rightarrow (g^2 N_c/\pi)\pi^2$ and $m_n^2 \rightarrow (g^2 N_c/\pi)\pi^2 n$, we recover the asymptotic result (8). We can reverse the argument by saying that in order to reproduce $\ln Q^2$ dependence in (8), the residues f_n^2 must go to the definite constant $\sim (m_{n+1}^2 - m_n^2)$ for large n (this result can be considered as the strict constraint of duality and the dispersion relations).

Now we want to repeat this analysis for the model we are interested in, namely, for QCD_2 with adjoint matter. As is known, the most important difference from t'Hooft model is that the bound states may contain, in general, *any* number of quanta. In other words, pair creation is not suppressed even in the large N limit. The problem becomes more complicated, but much more interesting, because pair creation imitates some physical gluon effects.

We consider the following correlator analogous to (7):

$$i \int dx e^{iqx} \langle 0|T\{\frac{1}{N_c} Tr \bar{\Psi}\Psi(x), \frac{1}{N_c} Tr \bar{\Psi}\Psi(0)\}|0\rangle = P_2(Q^2), \quad (11)$$

where $\bar{\Psi} = \Psi^T \gamma_0$ and label P_2 shows the number of partons of external source $\bar{\Psi}\Psi(x)$; the factor $1/N_c$ is included in the definition of the external current in order to make the right hand side of the equation independent of N . In the large Q^2 limit the leading contribution to correlation functions is given by the same diagram shown in Fig. 1a). The result is

$$P_2(Q^2 \rightarrow \infty) = -\frac{2}{2\pi} \left(\frac{1}{2} \cdot 2 \right) \ln Q^2. \quad (12)$$

The additional factor 2 in front of this expression comes from the two options in calculation of Tr and is related to the Z_2 symmetry mentioned in [7]. Besides that, we separate the trivial expression $\frac{1}{2} \cdot 2 = 1$ in parenthesis into two parts. The first factor $\frac{1}{2}$ is related to the Majorana nature of the fermions in comparison with Dirac's fermions in 't Hooft model. The second factor 2 which exactly compensates the first one, is related to the normalization of Ψ field (we follow the notation of [6]).

Now the problem arises. In the t'Hooft model we definitely knew that only 2-particle bound states contribute to the corresponding correlation function. This is not true any more for the model under consideration and any states, in general, may contribute to P_2 . Without any small parameter at hand the task of analyzing the problem seems hopeless. However, one can see that in the large Q limit small parameters do arise.

The *key* observation is as follows: any pair creation effects are suppressed by a factor $g^2 N_c / Q^2$ because of the dimensionality of the coupling constant in two dimensions (in a big contrast with real 4-dimensional QCD). Besides that, the quark mass term produces the analogous small factor m_q^2 / Q^2 and can be neglected as well. Thus, the information about highly excited states which provide the $\ln Q^2$ dependence at large Q^2 can be obtained exclusively from the analysis of the correlation function at large Q^2 . In particular, the mass of highly excited bound states should not depend on quark mass m_q (it gives small corrections to the leading $\ln Q^2$ behavior). With these remarks in mind we suggest the following pattern which saturates the correlation function (11, 12):

$$\begin{aligned} \langle 0 | \frac{1}{N_c} \bar{\Psi} \Psi | n_1 \rangle &= \sqrt{\frac{(m_0^2 \pi^2)}{\pi}} f_{n_1}, \quad n_1 \gg 1, \quad n_1 \in 2Z \\ m_{n_1}^2 &= m_0^2 \pi^2 n_1, \quad f_{n_1}^2 = 1, \quad m_0^2 = \frac{2g^2 N_c}{\pi}. \end{aligned} \quad (13)$$

In this formula we assume only one thing, namely that the mass spectra for massive states in QCD_2 coupled to fermions in the adjoint and fundamental representations are alike, and the highly excited states look like those in the 't Hooft model $m_n^2 \sim n$. The dispersion relations state in this case that the matrix elements (13) do not depend on excitation number n . Indeed, the only difference from the 't Hooft model is the doubling of the strength of the

interaction $g^2 \rightarrow 2g^2$ [6], and the additional degeneracy Z_2 , mentioned above (12). The formula for the correlation function P_2 (12) with the pattern (13) can easily be recovered:

$$P_2(Q^2 \rightarrow \infty) = \frac{2}{\pi} \sum_{n_1=0,2,4,\dots} \frac{(m_0^2 \pi^2) f_{n_1}^2}{m_{n_1}^2 + Q^2} \rightarrow \frac{2}{2\pi} \ln\left(\frac{\Lambda^2}{Q^2}\right), \quad (14)$$

where Λ^2 is the ultraviolet cutoff and related to the subtraction in the dispersion integral (instead one can consider $P_2(Q^2) - P_2(0)$ which is finite and does not depend on Λ^2 at all). As before, any corrections to the asymptotic expressions (13), like $f_{n_1}^2 = 1 + 0(1/n_1)$, $m_{n_1}^2 = m_0^2 \pi^2 n_1 + 0(m_q)$ produce some power corrections $\sim 1/Q^2$ and they are not interesting at the moment.

Thus we see the linear spectrum $m_n^2 \sim n$ unambiguously means that the residues f_{n_1} , $n_1 \gg 1$ do not depend on n_1 , excitation number, for large n_1 . This important consequence of the dispersion relations and duality will be used heavily in our following discussions.

We would like to repeat this analysis for the states with arbitrary number of $\Psi(x)$ fields. To do so, let us introduce the currents which create the $2k$ -parton states (as will be discussed in the next section, the mixing between different numbers of partons is not small. Thus, our term for the currents, creating the $2k$ -parton states, should be considered as convention.):

$$J_k(x) = \frac{1}{N_c^k} \text{Tr}(\bar{\Psi}(x) \Psi(x))^k, \quad \langle 0 | J_k | n_{\{k\}} \rangle = \sqrt{\frac{(m_0^2 \pi^2)}{\pi}} \sqrt{\left(\frac{m_0^2 \pi^2}{8\pi^2}\right)^{k-1}} f_{n_{\{k\}}}. \quad (15)$$

We introduced some numerical factors in the right hand side of this formula for future convenience. Let us consider the following correlation function given by the diagram of Fig.2:

$$i \int dx e^{iqx} \langle 0 | T \{ J_k(x), J_k(0) \} | 0 \rangle = P_{2k}(Q^2). \quad (16)$$

As before we can calculate the asymptotic limit of $P_{2k}(Q^2)$ at $Q^2 \rightarrow \infty$, in which quark mass can be neglected, and, after taking into account only leading term in $1/N_c$ expansion, get the expression:

$$P_{2k}(Q^2 \rightarrow \infty) = 2k \int d^2x e^{iqx} \left[\text{Tr} \left(\frac{\hat{x}}{2\pi x^2} \right) \left(\frac{\hat{x}}{2\pi x^2} \right) \right]^k = (-1)^k \frac{k}{\pi} \left(\frac{1}{8\pi^2} \right)^{k-1} \frac{(Q^2)^{k-1} \ln Q^2}{(k-1)!(k-1)!} \quad (17)$$

where $\hat{x} = x_\mu \gamma_\mu$.

The $(Q^2)^{k-1} \ln Q^2 / (k-1)!(k-1)!$ dependence in this equation is the most important part. We have to find the mass spectrum which will reproduce this large Q^2 behaviour. Of course one can not define the pattern of the mass

spectrum based only on the dispersion integral, so we have to make some assumptions again. First of all we again make assumption about the linearity of the spectrum which is based on numerical results [6], [8]. Using this assumption one can immediately see that each mass level has to be degenerate and our k -parton has to be classified by some quantum numbers n_1, n_2, \dots . In principle one can imagine that for highly excited states the formfactors $f_{n_{\{k\}}}$ depend on the quantum number and in this case the dispersion integral will not provide sufficient information to calculate the asymptotic behaviour of the spectrum. So we make the second assumption that the constants $f_{n_{\{k\}}}$ do not depend on $n_{\{k\}}$ in the limit $n_{\{k\}} \rightarrow \infty$. This assumption is close in spirit to the old ideas of “hadron democracy”. It is hard to imagine any reasonable pattern of strong dependence of formfactors $f_{n_{\{k\}}}$ on quantum numbers $n_{\{k\}}$, and it is unclear why some excited states should be produced more strongly than others, but for now we have no any rigorous proof. In any case these are the assumptions we are making here ⁴.

Thus our highly excited k -parton states are classified by the $n_{\{k\}} = n_1, n_2, \dots, n_k$ different numbers. Furthermore, the dependence of $m_{n_{\{k\}}}^2$ on n_i is linear (one additional argument in favor of the linear spectrum is the equivalence of the massive sectors in QCD_2^{adj} and QCD_2^{fund} with N_c flavours, according to [9]). The constants $f_{n_{\{k\}}}$ do not depend on $n_{\{k\}}$ in the limit $n_{\{k\}} \rightarrow \infty$. It is clear, that these $n_{\{k\}}$ numbers should be identified with the excitation numbers of the k partons. Using the symmetry under: $n_1 \leftrightarrow n_2 \leftrightarrow n_3 \dots \leftrightarrow n_k$ we conclude that $m_{n_{\{k\}}}^2 = m_0^2 \pi^2 (n_1 + n_2 + \dots + n_k)$. This formula was obtained previously [7], but we want to emphasize here that it has a much more general origin which is not related to the particular solution of the model suggested in [7]. The dispersion relations can now be written in the following form:

$$P_{2k}(Q^2) = \frac{(m_0^2 \pi^2)}{\pi} \left(\frac{m_0^2 \pi^2}{8\pi^2} \right)^{k-1} \sum_{n_1, n_2, \dots, n_k=0,2,4,\dots} \frac{f_{n_{\{k\}}}^2}{(m_{n_{\{k\}}}^2 + Q^2)} \Rightarrow \quad (18)$$

$$\frac{(m_0^2 \pi^2)}{\pi} \left(\frac{m_0^2 \pi^2}{8\pi^2} \right)^{k-1} \sum_{N \gg 1}^{\infty} \frac{f_k^2 G_k(N)}{m_0^2 \pi^2 N + Q^2} = (-1)^k \frac{k}{\pi} \left(\frac{1}{8\pi^2} \right)^{k-1} \frac{(Q^2)^{k-1} \ln Q^2}{(k-1)!(k-1)!}$$

where we introduced the notation f_k for the constants which might depend on the number of partons k , but not on their excitation numbers $n_{\{k\}}$: $f_{n_{\{k\}}} \rightarrow f_k$, $n_{\{k\}} \rightarrow \infty$. We also introduced the factor of degeneracy $G_k(N = n_1 + \dots + n_k)$ for the bound states of $2k$ partons with the given mass $m^2 = m_0^2 \pi^2 N$ for highly excited states. Now, the only way to satisfy the dispersion relation at asymptotically large Q^2 (after an appropriate number of subtractions) is

⁴ We also checked that these assumptions are selfconsistent with the large Q^2 asymptotics of some three- and four-point correlation functions of currents under consideration.

to put

$$G_k(N) = \frac{2kN^{k-1}}{(k-1)!(k-1)!f_k^2} \frac{1}{f_k^2}, \quad N \gg 1, \quad (19)$$

where the factor 2 is related to the Z_2 symmetry mentioned above. In this case, by taking into account that only the even states contribute to (18) we exactly reproduce eq.(18).

Having at hand the expression for $G_k(N)$ it is very easy to calculate the total number of excited $2k$ -parton states with mass less than $\mathbf{M}^2 = m_0^2\pi^2\mathbf{N}$, $\mathbf{N} \gg 1$:

$$\sum_{N \sim 1}^{\mathbf{N}} G_k(N) \simeq \frac{2\mathbf{N}^k}{(k-1)!(k-1)!f_k^2} \frac{1}{f_k^2}, \quad \mathbf{N} \gg 1. \quad (20)$$

Our last step is the summation over all $2k$ bound states. To perform such a calculation we need to make some assumption about the k -dependence of the matrix elements f_k . Such information can be obtained scientifically only from explicit dynamical calculations; note that the dispersion relations can constrain the product $f_k^2 \cdot G_k(N)$, but not f_k , G_k separately. However, we can parametrize the k -dependence in the following general way:

$$\frac{1}{f_k} = c^k k^\alpha, \quad n_k \rightarrow \infty \quad (21)$$

The parameter c in this formula comes from the normalization of the wave function and a nonzero α may arise because of an interaction between different pairs of partons. Now the sum we are interested in

$$\int^{\mathbf{M}} \rho(m) dm \sim \sum_{k=1} k^{2\alpha+2} \frac{(c\mathbf{M}/m_0\pi)^{2k}}{k!k!} \quad (22)$$

can be estimated by using a saddle point approximation where the extremum of the sum is at $k = (c\mathbf{M}/m_0\pi)$. After some algebra one gets

$$\rho(m) \sim m^{2\alpha+3/2} \exp \left[c \left(\frac{\sqrt{2}}{\pi} \sqrt{\frac{\pi}{g^2 N_c}} \right) m \right]. \quad (23)$$

This is the main result of our paper - we demonstrate that the Hagedorn spectrum is determined in terms of the asymptotics of the matrix elements f_k . The parameters c and α in (21) which describe these asymptotics define the Hagedorn temperature $T_H \sim 1/c$ as well as parameter $a = -2\alpha - 3/2$. If $\alpha > -3/2$ then $a = -2\alpha - 3/2 < (D+1)/2 = 3/2$ which means that the Hagedorn temperature is the limiting temperature of the system.

We would like to go further in order to make some specific assumption about the k dependence (21). The model under consideration is a theory with Majorana fermions. This makes some difference in calculating the diagramm

Fig.2; each extra loop gives an extra factor $1/2$ in comparison with the Dirac case. We believe that this extra factor comes into the definition of the matrix elements f_k . Thus, we *assume* that the transition from Dirac fermions to Majorana fermions gives an additional factor $f_k^2 \sim \frac{1}{2^k}$. Let us repeat again: we can not prove or disprove the appearance of this additional element in comparison with the case of Dirac fermions in the 't Hooft model. It can be proven only as the result of the dynamical calculations. Nevertheless, assuming the existence of this additional factor and summing over all $2k$ states in formula (20) we get (we use the standard formula for the expansion of Bessel function $I_0(z) = \sum_{k=0} \frac{(z/2)^{2k}}{(k!)^2}$):

$$\int^{\mathbf{M}} \rho(m) dm = 2 \frac{\mathbf{M}^2}{m_0^2 \pi^2} \sum_{k=0} \frac{(\mathbf{M}/m_0 \pi)^{2k} 2^k}{(k!)^2} = 2 \frac{\mathbf{M}^2}{m_0^2 \pi^2} I_0(z), \quad (24)$$

where

$$z = \frac{2}{\pi} \sqrt{\frac{\pi}{g^2 N_c}} \mathbf{M}$$

We expect that the corrections to this formula from the non-highly excited states as well as from mixing with different parton states might give some power corrections (or even change the order of the Bessel function I_ν). However, this does not effect the asymptotic behaviour $z \rightarrow \infty$, which is one and the same for all Bessel functions $I_\nu(z) \rightarrow e^z / \sqrt{2\pi z}$ and gives us the spectrum

$$\rho(m) \sim m^{3/2} \exp(\beta_H m), \quad \beta_H = \frac{2}{\pi} \sqrt{\frac{\pi}{g^2 N_c}} \approx 0.64 \sqrt{\frac{\pi}{g^2 N_c}} \quad (25)$$

This formula is in agreement with the general expression (23) where $c = \sqrt{2}$, $\alpha = 0$. As we mentioned above, the parameter α describes the interaction between different pairs of partons. We have no rigorous arguments that $\alpha = 0$. However, for asymptotically high $n_k \gg 1$ it is very unlikely that the interactions change the magnitude of α . One type of argument is based on consideration of, let us say, a four-parton system (i.e. $k = 2$) with large n_1 and n_2 but with $n_1 \gg n_2$. The interaction between these two subsystems is suppressed again by factor $\frac{N_c g^2}{Q^2}$. Thus we do not expect that the interaction will change the asymptotic behavior of the correlation function. Therefore, it can not contribute to α . However these arguments may be spoiled by some complicated behavior of the multiparton wave function $\Phi(x_1, \dots, x_{2k})$ at small x .

Let us note that the analytical result (25) for the inverse Hagedorn temperature is in fair agreement with the numerical results [6], where the value $(0.7 \div 0.75)$ has been obtained (compare to our factor $\frac{2}{\pi} \simeq 0.64$).

3. Condensate $\langle 0 | \text{Tr}(\bar{\Psi}\Psi) | 0 \rangle$.

We already mentioned that we have a qualitative agreement with the numerical results [6] regarding the Hagedorn temperature. Another qualitative

numerical result which was mentioned in [6] is that the wave function of a typical excited state is a complicated mixture of components with different parton numbers.

Apparently, such a mixing is very difficult to explain by analysing the correlation functions at large Q^2 (the method we use throughout this paper). Indeed, if we consider the non-diagonal correlator analogous to (11), but with different number of constituents ($2 \rightarrow 4$, as example):

$$i \int dx e^{iqx} \langle 0 | T \{ \frac{1}{N_c^2} \text{Tr}(\bar{\Psi}\Psi(x))^2, \frac{1}{N_c} \text{Tr}\bar{\Psi}\Psi(0) \} | 0 \rangle = P_{2 \rightarrow 4}(Q^2), \quad (26)$$

one could naively think that this correlation function is strongly suppressed because of extra powers of either g^2/Q^2 or m_q^2/Q^2 (see Fig.3). The naive interpretation in this case would be that the highly excited states saturating an appropriate dispersion relation are pure states with definite number of partons. Any mixing $2 \text{ partons} \rightarrow 4 \text{ partons}$ is highly suppressed. Such a conclusion is in severe contradiction with the numerical results [6]. What is wrong?

We see only one (but very natural) resolution of this puzzle. Namely, we **assume** that the theory possesses the condensate

$$\langle 0 | \text{Tr}(\bar{\Psi}\Psi(x)) | 0 \rangle = \mu N_c^2 \neq 0. \quad (27)$$

In that case the suppression will be gone (here μ is some dimensional number, proportional in the limit $m_q \rightarrow 0$ to m_0). Indeed, the asymptotically leading (at $Q^2 \rightarrow \infty$) contribution to eq.(26) will be determined by the diagramm Fig.4 and not Fig.3. Thus, it is equal to

$$P_{2 \rightarrow 4}(Q^2) \sim i \int dx e^{iqx} \langle 0 | T \{ \frac{1}{N_c^2} \text{Tr}(\bar{\Psi}\Psi(x))^2, \frac{1}{N_c} \text{Tr}\bar{\Psi}\Psi(0) \} | 0 \rangle \sim \frac{\langle 0 | \text{Tr}\bar{\Psi}\Psi(0) \rangle}{N_c} \cdot \frac{\delta_j^i}{N_c} \cdot i \int dx e^{iqx} \langle 0 | T \{ \frac{1}{N_c} \bar{\Psi}_i^k \Psi_k^j(x), \frac{1}{N_c} \text{Tr}\bar{\Psi}\Psi(0) \} | 0 \rangle. \quad (28)$$

The right hand side of this expression coincides with the correlation function (11) and is not suppressed. Once again, we have assumed that the chiral condensate is not zero in this theory. Having made this assumption, we can explain theoretically the numerical results [6] on strong mixing of components with different parton numbers.

Formula (28) suggests the following pattern of saturation of the corresponding dispersion relation. Let us denote the state $|X\rangle$ as the eigenstate with mass m_X which contributes to the asymptotic behavior of the correlation function (11)(these states are not necessary two particle states). The corresponding matrix element will be denoted as f_X :

$$\langle 0 | \frac{1}{N_c} \text{Tr}(\bar{\Psi}\Psi) | X \rangle = f_X. \quad (29)$$

Now, from the asymptotic expression (28) one can see that the dispersion relation corresponding to correlator (28) will be **automatically satisfied** if the appropriate matrix element can be expressed in terms of f_X in the following way:

$$\langle 0 | \frac{Tr(\bar{\Psi}\Psi)^2}{N_c^2} | X \rangle \sim \langle 0 | \frac{Tr(\bar{\Psi}\Psi)}{N_c} | X \rangle \frac{\langle 0 | Tr(\bar{\Psi}\Psi(0)) | 0 \rangle}{N_c^2} \sim f_X \frac{\langle 0 | Tr(\bar{\Psi}\Psi(0)) | 0 \rangle}{N_c^2} \quad (30)$$

An analogous expression can be written for an arbitrary non-diagonal correlator. Saturation of the corresponding dispersion relation will occur automatically for all such cases. The reason is that the calculation of a nondiagonal correlation function (like (28)) using the factorization of the $\langle 0 | Tr(\bar{\Psi}\Psi(0)) | 0 \rangle$ term (see Fig.4) and the evaluation of the matrix element (like(30)), by factorizing the same expression $\langle 0 | Tr(\bar{\Psi}\Psi(0)) | 0 \rangle$, is **one and the same** procedure.

The moral of this discussion is very simple: in order to have a mixing (to be in agreement with the numerical calculations) we have to have some non-suppressed off-diagonal correlation functions. This goal can be achieved only by assuming a non-zero magnitude for the condensate $\langle 0 | Tr(\bar{\Psi}\Psi(0)) | 0 \rangle$. In other words: the non-zero magnitude for the chiral condensate and the phenomenon of mixing of components with different parton numbers are two sides of the same coin. The dispersion relations connect these two, apparently different, phenomena.

In what follows we give some arguments that such a condensation is a very natural phenomena in the theory under consideration and it is very likely to happen. To avoid any confusion, we would like to note from the very beginning of these discussions, that this condensate does not break down any continuous symmetry; thus, no goldstone boson appears as a consequence of the condensation.

It has been known for a long time [20] that gauge theory with the matter in the adjoint representation has unique topological properties. Namely, in contrast with the standard QCD, the gauge group G is $SU(N)/Z_N$ rather than just $SU(N)$ ⁵. In the formal terms it means that the first homotopy group

$$\pi_1\left(\frac{SU(N)}{Z_N}\right) = Z_N$$

is not trivial, so that there are N topologically nonequivalent sectors. This is a new classification which goes together with the standard topological analysis.

These arguments are very general, and thus they are not specific to the space-time dimensionality of the theory. In particular, the realization of

⁵ Z_N in this formula represents the elements of the center of the group. As is known, the adjoint fields are not transformed under the action of the elements of the center.

this extra symmetry in four dimensional Yang-Mills theory leads to the so-called toron solutions [21] which may be responsible for the chiral condensate $\langle 0|\lambda^2|0\rangle$ in supersymmetric Yang-Mills theory[22], [23] ⁶. The vacuum condensate

$$\langle 0|\lambda^2|0\rangle_k \sim \exp(\frac{i2\pi k}{N})$$

in this model labels the different vacuum states marked by the integer number k . The physics in all these vacua is the same and the number of these vacuum states is N in agreement with the additional classification mentioned above and the value of the Witten index which equals N [24].

A very similar phenomenon takes place in two dimensional supersymmetric CP^{N-1} models. In this case, again, there is a condensate [23] which classifies N different vacuum states $\langle 0|\lambda^2|0\rangle_k \sim \exp(\frac{i2\pi k}{N})$.

Therefore, it is natural to expect that QCD_2 with the gauge group $SU(N)$ and matter in the adjoint representation has a kind of discrete

$$\theta = \frac{2\pi k}{N}$$

vacua. Indeed in the ref. [25] it was shown that the non-trivial topological classification related to $\pi_1(SU(N)/Z_N) = Z_N$ leads to N different vacuum states. Therefore it is natural to think that in the presence of adjoint matter these vacuum states are classified by the vacuum condensate. The recent explicit calculations of the condensate for $SU(2)$ [26] and $SU(3)$ [27] gauge groups support this conjecture.

Let us repeat again that we do not attempt to calculate the condensate in the present paper. Instead we make a conjecture about the existence of this condensate based on the previous analysis of the model. In this case we are in a position to explain the numerical results about the spectrum in the model [6]. We could reverse our arguments by saying that the complicated mixing discovered in the numerical calculation can be explained from the theoretical point of view only if the theory possesses a chiral condensate.

We believe that the condensation of the fermion adjoint field in the theory is a very important issue. Thus, we would like to present some independent arguments in favour of this conjecture.

The existence of the condensate $\langle 0|Tr\bar{\Psi}\Psi(0)|0\rangle$ in $QCD_2(N_c)$ with Majorana fermions can be seen using the bosonized version of the theory[26]. As is known, there is a one to one correspondence between the original Majorana fermion bilinear $\bar{\Psi}^a\Psi^b$, and some other boson field given by an orthogonal matrix Φ^{ab} [28]. The precise correspondence

$$\bar{\Psi}^a\Psi^b = \mu\Phi^{ab}, \quad a, b = 1, 2, \dots, N_c^2 - 1 \quad (31)$$

⁶Let us recall that this model contains the standard gluon fields as well as the adjoint matter fermions, so-called gluino fields λ .

depends on the regularization procedure and normalization convention. If the standard normalization $\langle 0|\Phi^{ab}|0\rangle = \delta^{ab}$ is chosen, then $\langle 0|\bar{\Psi}^a\Psi^a|0\rangle = \mu N_c^2$ with $\mu \sim m_0$. Thus, in the bosonized framework we should not be surprised by appearance of a condensate (it is a very natural phenomenon). Rather, we should be surprised by the fact that sometimes (e.g. QCD_2 with fundamental fermions and finite N_c) the condensate is zero.

To clarify the last statement and to demonstrate the difference between Dirac and Majorana fermions, let us consider $QCD_2(N_c)$ with fundamental fermions for arbitrary N_c (not necessary the 't Hooft model with $N_c \rightarrow \infty$). The standard bosonisation rules for color singlet operators (only this part is important for the following discussions) have the form:

$$\bar{\psi}(1 \pm \gamma)\psi \rightarrow \exp(\mp i2\sqrt{\pi}\theta(x)), \quad \bar{\psi}i\partial_\nu\gamma_\nu\psi \rightarrow \frac{N_c}{2}(\partial_\nu\theta(x))^2 \quad (32)$$

Taking into account this formula one can calculate the following correlation function (which actually determines the condensate) at large distances[11]:

$$\langle 0|T\{\bar{\psi}(1 + \gamma_5)\psi(x), \bar{\psi}(1 - \gamma_5)\psi(0)\}|0\rangle \sim \exp(i\frac{2\pi}{N_c}\Delta(x)) \sim x^{-\frac{1}{N_c}}. \quad (33)$$

Here $i\pi\Delta(x) \sim -\ln(x^2)$ is the propagator of the **massless** scalar field $\theta(x)$. The fact that the field $\theta(x)$ remains massless even when interactions are taken into account was crucial in the derivation of (33). This fact is clearly related to the chiral symmetry $\theta \rightarrow \theta + const$ of the original Lagrangian and the absence of an anomaly in the singlet channel, which could break this symmetry explicitly (like in the Schwinger model). Besides that, in deriving (33) we took into account that in the infrared region ($x \rightarrow \infty$) only the term of lowest dimension in the Lagrangian $\sim \frac{N_c}{2}(\partial_\nu\theta(x))^2$ is important.

This derivation explicitly demonstrates the difference between Dirac and Majorana fermions: in the former case there is a symmetry which can not be broken and it prevents the condensation; in the latter case there is no symmetry which could prevent the Majorana field from condensation. Formula (33) also explains the behavior of the 't Hooft model in the large $N_c \rightarrow \infty$ limit, where the correlation function (33) goes to a constant. Such a behavior together with the cluster property at $x \rightarrow \infty$ implies the existence of the condensate at $N_c = \infty$ in agreement with the explicit calculation [11]. At the same time, for any finite but large N_c the correlator falls off very slowly demonstrating the BKT- behavior [18] with no contradiction of the Coleman theorem.

We believe that we have convinced the reader that the condensation is an unavoidable feature of the model (we do not know how to calculate it, but that is a different issue). If this is correct, let us formulate the following question: what kind of field configurations are responsible for this condensation? The honest answer to this question – we do not know. However the important property of the large N_c limit is that the expectation value of

a product of any invariant operators reduces to their factorized values[29]. Thus, one could expect that the **classical master field** saturates this condensate. However, we do not know how this might be explicitly realized. Let us note that a similar situation occurs place in the 't Hooft model, where the mixed vacuum condensates with arbitrary number of gluon insertions $\langle 0|\bar{q}(g\epsilon_{\mu\nu}G_{\mu\nu})^n q|0\rangle \sim M_{eff}^{2n}\langle 0|\bar{q}q|0\rangle$ can be explicitly calculated [19]. Again, one could interpret such a factorization property as reminiscent of the master field at large N_c .

One more issue which is related to the condensation of the Majorana field is as follows. We argued earlier that the non-zero result for a non-diagonal correlation function analogous to (26) is due to the non-zero condensate $\langle 0|Tr\bar{\Psi}\Psi(0)|0\rangle$. At the same time such a correlation function describes the **mixing** between different numbers of partons. This mixing is order of one (not small!) just because the corresponding correlation function is not suppressed! The same is true for an arbitrary external current with an arbitrary number of $(\bar{\Psi}\Psi)^k$ in it. Thus, we arrive at the following conclusion: The condensation of the fermion field is **responsible** for the complicated mixture of components with different parton numbers in hadronic states.

Furthermore, because of the factorization of $\langle 0|Tr\bar{\Psi}\Psi(0)|0\rangle$ in the course of the calculation of hadronic matrix elements one could expect that the probability to have an additional quark pair is not small (the direct consequence of such a calculation is the absence of any suppression if one adds a few more partons to the system). Thus, the momentum which is carried by a given parton in a hadron, is getting smaller and smaller when we inject (without suppression) additional partons ⁷.

Thus, we arrive at the next conclusion: The condensation of the fermion field leads to the Feynman-Bjorken picture of **wee** partons [30]. Such a connection between condensation of $\langle 0|Tr\bar{\Psi}\Psi(0)|0\rangle$ and an infinite number of constituents near zero momentum has been known for a long time(see the recent review talk [31] delivered at the Light Cone meeting by one of the pioneers of the subject). The only comment we would like to make here is as follows.

The nonzero condensate means that only part of the sea quark distribution gets Lorentz contracted when the hadron is boosted, while another component looks the same in all frames. Fig.4 explains this statement in terms of correlation function. Dispersion relations transform this statement into one about hadrons. In the four dimensional world this would mean that the hadron does not become a “pancake” as $Q \rightarrow \infty$. We believe that the model under consideration is a model where:

a). the light cone quantization is already performed;

⁷Actually our arguments, based on a correlation function consideration, can be applied only for highly excited states. Only those states saturate the dispersion integral. Only for those states can information about the correlation function be transformed into knowledge of hadronic states.

- b). the mixing is believed to be large;
- c). the condensate $\langle 0|Tr\bar{\Psi}\Psi(0)|0\rangle$ is not zero;
- d). the Feynman-Bjorken picture of **w** partons emerged.

Thus it is a perfect model to explore the connections, mentioned above, between these apparently different problems.

4. Conclusion.

Let us note that the behaviour of the system at finite temperature T is equivalent in the euclidian path integral formalism to the behaviour of the same system on the cylinder $S^1 \times R^1$ with radius $R = (2\pi T)^{-1} = (\beta/2\pi)$. One can consider now compact directions S^1 as a space directions and R^1 as an euclidian time.

Then our results mean that there is a minimal radius $R_c = \beta_H/2\pi$ - however one can ask what will happen if we consider QCD with adjoint matter living on a circle S^1 with radius $R < R_c$. Let us note that we can not reach this small radius smoothly starting from large R because the energy will diverge as $E(R) \sim 1/(R - R_c)^{3+2\alpha}$. This is nothing but the Casimir energy which becomes singular at R_c and prevents the circle from being squeezed further.

Now in the case when $R < R_c$ from the very beginning we definitely have no hadron spectrum - the properties of this phase are obscure. It is known, however, that the spontaneous breaking of Z_N symmetry in this model is an unusual one [32] - contrary to common belief two different Z_N phases can not coexist in the same space. We also would like to mention here that it may be that two theories with the same massive spectra, like adjoint QCD_2 and QCD_2 with N_c fundamental fermions (see [9]) may have different high temperature (small compactification radius) properties - indeed the first theory has a Z_N degeneracy of the ground state in the high-temperature phase, and the second does not.

It is interesting to understand what type of string theory will reproduce the behaviour of 1 + 1 QCD at high temperatures and/or small radii R , in particular what will be the spectrum of the winding modes, which becomes tachyonic [33] at the point of the phase transition. Let us note that in this theory besides the winding modes corresponding to pure gluon non-contractible loops $L = \frac{1}{N_c} \text{tr } P \exp \left(i \oint_S A_\mu dx^\mu \right)$ there are more complicated objects with quarks

$$L(x_1, \dots, x_n) \sim$$

$$\text{tr} \left(P \Psi(x_1) \exp \left(i \int_{x_1}^{x_2} A_\mu dx^\mu \right) \Psi(x_2) \dots \Psi(x_n) \exp \left(i \int_{x_n}^{x_1} A_\mu dx^\mu \right) \right) \quad (34)$$

where x_1, \dots, x_n are the points on a noncontractible contour S_1 . In principle $L(x_1, \dots, x_n)$ may decay into usual hadron and pure gluon winding mode L ,

however it may be that the most unstable winding modes are of the type (34) and further investigation is necessary to answer this and many other questions about the high T or small R phase of this theory.

In the short term there are a number of areas in which progress can be made. First, a numerical calculation of the condensate can be done in the same way as in the t'Hooft model. Besides that we believe that the information about the condensate is hidden in the numerical calculation (already completed) [6] and luckily it can be extracted from the data bank. The last (but not the least) issue is the exploration of the following relations:

Masterfield $\Leftrightarrow \langle 0 | \text{Tr} \bar{\Psi} \Psi(0) | 0 \rangle \Leftrightarrow$ *Mixed states* \Leftrightarrow *wee partons* \Leftrightarrow *quark model* \Leftrightarrow *Light cone framework* \Leftrightarrow *Zero modes*.

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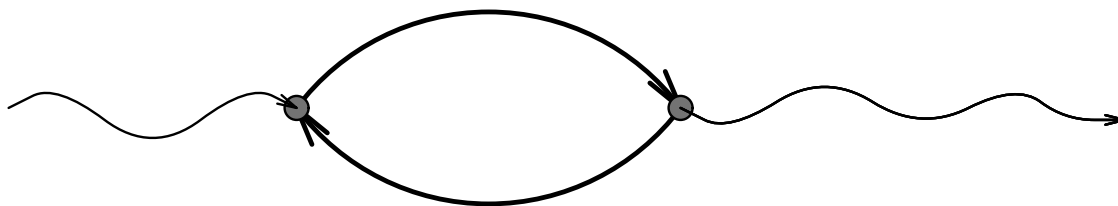


Fig. 1

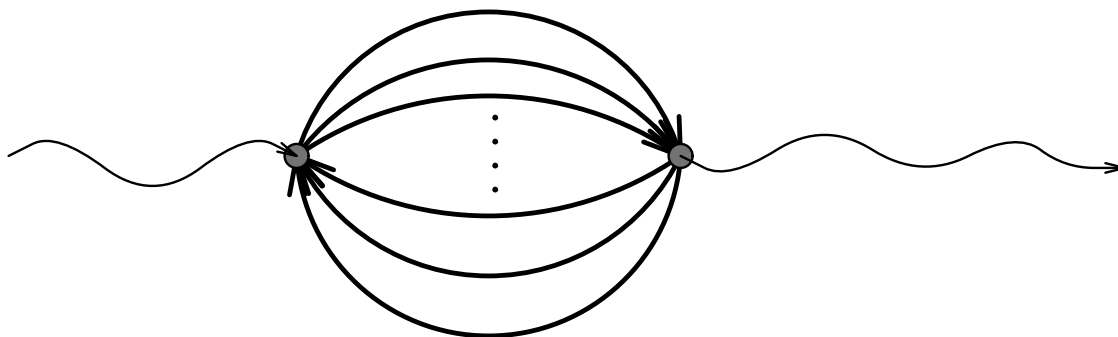


Fig. 2

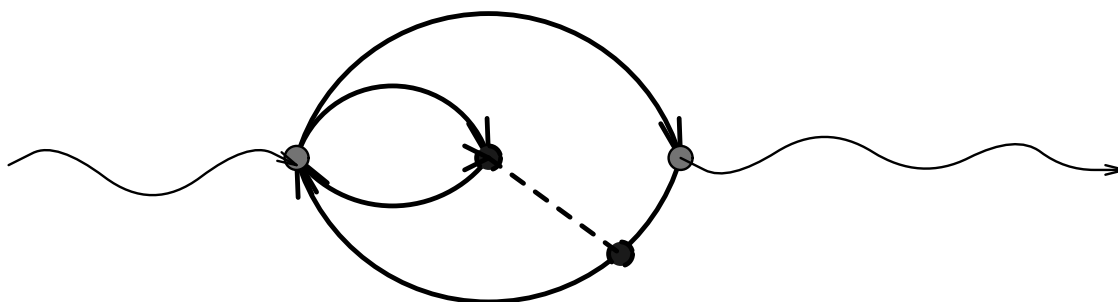


Fig. 3

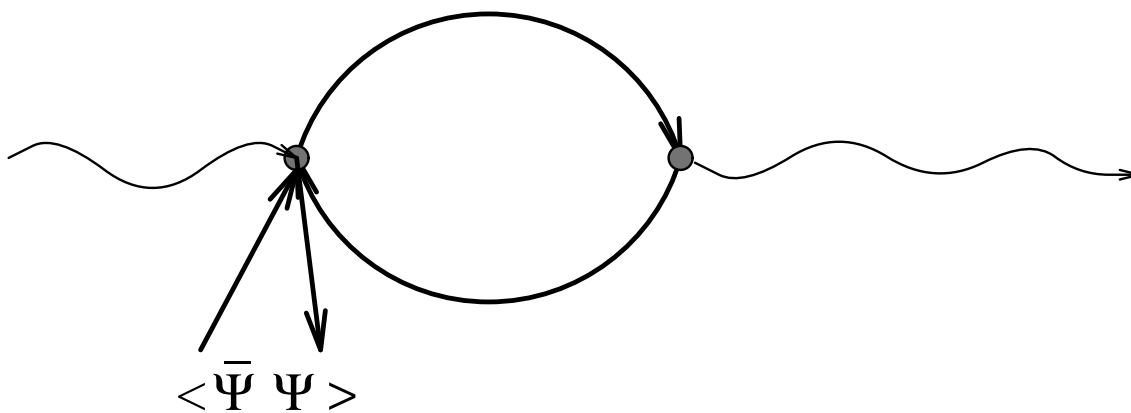


Fig. 4